U S A Mathematical Talent Search

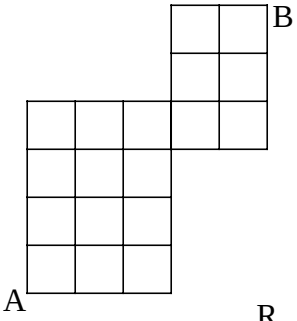
PROBLEMS
Round 2 - Year 11 - Academic Year 1999-2000

1/2/11. The number N consists of 1999 digits such that if each pair of consecutive digits in N were viewed as a two-digit number, then that number would either be a multiple of 17 or a multiple of 23. The sum of the digits of N is 9599. Determine the rightmost ten digits of N.

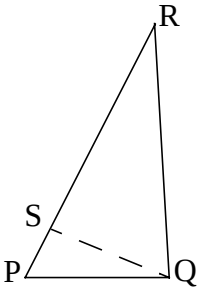
2/2/11. Let C be the set of non-negative integers which can be expressed as $1999s + 2000t$, where s and t are also non-negative integers.

(a) Show that 3,994,001 is not in C.

(b) Show that if $0 \le n \le 3{,}994{,}001$ and n is an integer not in C, then $3{,}994{,}001 - n$ is in C.

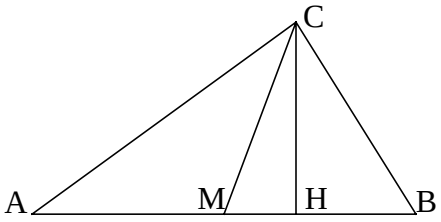


3/2/11. The figure on the right shows the map of Squareville, where each city block is of the same length. Two friends, Alexandra and Brianna, live at the corners marked by A and B, respectively. They start walking toward each other's house, leaving at the same time, walking with the same speed, and independently choosing a path to the other's house with uniform distribution out of all possible minimum-distance paths [that is, all minimum-distance paths are equally likely]. What is the probability they will meet?



4/2/11. In ΔPQR, PQ = 8, QR = 13, and RP = 15. Prove that there is a point S on line segment $\overline{PR}$, but not at its endpoints, such that PS and QS are also integers.

5/2/11. In ΔABC, AC > BC, CM is the median, and CH is the altitude emanating from C, as shown in the figure on the right. Determine the measure of $\angle MCH$ if $\angle ACM$ and $\angle BCH$ each have measure $17°$.



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Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet should be sent to the following address and **postmarked no later than November 15, 1999**. Each participant is expected to develop solutions without help from others.

USA Mathematical Talent Search
COMAP Inc., Suite 210
57 Bedford Street
Lexington, MA 02173